Superposition in Infiltration Modeling

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ABSTRACT

Simplified, physical models for calculating infiltration and ventilation in a single zone, usually calculate the air flows from the two natural driving forces (i.e., wind and stack effects) separately, and then use a superposition law to combine them. Similarly superposition laws may be used to ascertain the effects of mechanical systems on infiltration. In this report a general superposition law will be derived for combining wind, stack, and mechanical ventilation systems together. The superposition law will be derived using general principles of leakage distribution and air flow and will not depend on the details of a particular infiltration model. In the process of generating this law, a quantity called leakage distribution angle is developed to quantify the separation of areas of the building envelope which are subject to infiltration and exfiltration. The general superposition law is compared to other proposed superposition laws including those based on measured data, and the general law is shown to have strong explanatory power. Results are generated for typical buildings. The concept of fan addition efficiency is developed to determine the effectiveness of unbalanced (mechanical) ventilation systems at augmenting infiltration.

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NOMENCLATURE

B	AIM Superposition Constant [-]
C'	Shielding Coefficient [-]
f	Air Flow Ratio [-]
g	Acceleration of gravity [m/s^2]
H	Stack Height [m]
$\mathbf{K}$	Leakage Coefficient
n	Leakage Exponent [-]
P	(Air) pressure [Pa]
ΔP	Representative pressure drop across the envelope [Pa]
Q	Air flow [m^3/hr]
v	(Local) Wind Speed [m/s]
α	Quadrature Constant [-]
β	(Dimensionless) Neutral Level (when only stack effect operates) [-]
ε_+	Addition Efficiency [-]
ρ	Density (of air) kg/m^3
θ	Leakage Distribution Angle [-]

Subscripts indicate values associated with:

+	*in*filtration
-	*ex*filtration
1,2	any driving force
n	a natural driving force
w	wind effect
$\equiv$	wind striking a building face
$\neq$	wind striking diagonally
s	stack effect
f	(unbalanced) fan

INTRODUCTION

The calculation of infiltration-dominated ventilation usually requires the combination of wind-induced, temperature-induced, and mechanically-induced air flows. Complex models solve the problem by finding the pressure at each point on the envelope and then solving for the flow— modifying the internal pressure in order to satisfy the continuity equation.[1] Such an approach is very powerful, but may require inputs and computational requirements that may make it impractical. For many applications simpler models are desirable, even if less accurate.

In most simple (single-zone) models it is a relatively straight-forward problem to calculate the pressures-induced flow for a single one of the driving forces. Each of these three mechanism induce pressures across the envelope to drive the flow, but the spatial distribution of the pressure is different for each one of them. Although these pressures are additive on a point-by-point basis, the flows induced by those pressures are not. Combining these flows together in a similarly simple way is the topic of this report. Although general properties of the three mechanisms will be discussed, details such as fan curves, pressure coefficients, leakage distributions, etc. will not.

BACKGROUND

Infiltration is pressure-driven air flow through the envelope of the building, so it is important to understand the leakage properties in order to understand the infiltration. The leakage of the envelope is conventionally treated as a power law.[2] The measurement of leakage is usually performed with a technique called fan pressurization[3] wherein the fan flow induces a shift in the internal pressure:

$$Q_f = \mathbf{K} \Delta P_o^n \tag{1}$$

where the exponent $\frac{1}{2} \leq n \leq 1$ depending on the hydrodynamics of the leaks.

As can be seen from the appendix, the fan pressurization flow is the large fan limit of an unbalanced fan. $\mathbf{K}$ and the exponent, n, characterize the leakage; In addition to being measured from a fan pressurization test, they can be found from more advanced techniques[4,5] Although the envelope could have different exponents in different areas, we will assume that the exponent does not vary.

The exponent is a particularly important characteristic of the flow for both understanding the behavior and modeling it. If the exponent were unity, the modeling would be linear and relatively simple. For most buildings, however, the exponent is in the range $0.55 \leq n \leq 0.75$ with $n = 2/3$ being a typical value;[6] and the modeling becomes more complex. For this typical value of exponent, however, $\mathbf{K}$ becomes independent of temperature in normal situations (See ref. 2).

Each one of the three driving forces has a particular pattern of pressures induced across the envelope. From this pattern of pressures and the flow characteristics of eq. 1, the air flow from that driving force can be calculated, albeit differently for different models. Since combining the different driving forces on a point-by-point basis is more complex than the simple models can deal with, a superposition law that combines the individually calculated flows in a simple but robust way is needed.

REVIEW OF SUPERPOSTION METHODS

If the driving forces were independent and did not interact, a reasonable superposition law would simply add the flows algebraically. This pressure independent situation only occurs for the case of balanced supply and exhaust fans (e.g., an air-to-air heat exchanger). If there are both supply *and* exhaust fan operating simultaneously, the balanced part of the flow will add algebraically to the that air change calculated using the unbalanced part:

$$Q = Q_{balanced} + f(Q_s, Q_w, Q_f) \tag{2}$$

where the balanced part is the smaller of the supply and exhaust flows:

$$Q_{balanced} \equiv MIN\left[Q_{supply},\ Q_{exhaust} \right] \tag{3}$$

In the sections to follow the fan *driving force* will be due only to the unbalanced part of the fan flow:

$$Q_f \equiv \left| Q_{exhaust} - Q_{supply} \right| \tag{4}$$

which reduces to the net amount of exhaust (supply) flow.

Because the individual driving forces affect the pressure distribution, they interact and simple addition usually leads to significant overpredictions. Therefore, in general a sub-additive superposition law will be required.

One of the first simplified physical models of infiltration, the LBL model,[7,8] used the following superposition law:

$$Q^2 = Q_1^2 + Q_2^2 \tag{5}$$

This superposition law is called "*simple quadrature*" or "*LBL superposition*".

Using measured data Wilson and Pittman[9] have shown that this type of model captures much of the physical behavior. Using measurement and simulation for full-scale test structure, Mobile Infiltration Test Unit (**MITU**), Modera et al.[10] have shown that there can be an overprediction error on the order of 25% when the wind and stack effects are equal.

The LBL infiltration model assumes orifice flow and thus fixes the leakage exponent at one half. Various other efforts have attempted to generalize the model by using the measured exponent.[11] In some of these *Variable Flow Exponent* (**VFE**) models an *exponefied* version of simple quadrature is used to generalize the superposition rule:[12]

$$Q^{1/n} = Q_1^{1/n} + Q_2^{1/n} \tag{6}$$

We call this superposition law "*VFE superposition*".

Using a detailed simulation Modera and Peterson[13] have investigated both LBL and VFE superposition for the combination of wind effects and stack effects with and without the operation of mechanical exhaust. The specific example cited uses the configuration of MITU. They found that, in general, simple quadrature works better than the exponefied version and both may over-predict the total when there is no fan operation. Further, the deviation in simple quadrature is found to be a strong function of leakage distribution.

In order to mitigate the overprediction of the VFE superposition law, the NRC model[12,14] uses an ad-hoc correction factor:

$$Q = \left[0.8 f_n^{-0.1}\right] \left(Q_s^{1/n} + Q_w^{1/n}\right)^n \tag{7}$$

where:

$$f_n \equiv \frac{Q_{smaller}}{Q_{larger}} \leq 1 \tag{8}$$

When f_n becomes small enough (≈0.1) the term in brackets gets replaced by unity. This correction, therefore, always reduces the value relative to VFE superposition and has the biggest effect (≈20%) when the two flows are equal (i.e., $f_n \approx 1$). We call this law "*NRC superposition*".

Walker and Wilson[15] modify VFE superposition in an algebraically simpler method:

$$Q^{1/n} = Q_s^{1/n} + Q_w^{1/n} + B\, Q_w^{1/2n} Q_s^{1/2n} \tag{9.1}$$

From their data they have found that

$$B \approx -\frac{1}{3} \tag{9.2}$$

We call this law "*AIM superposition*".

General Quadrature

VFE (as well as LBL) superposition is based on assumption that the infiltration from each driving force can be treated as an effective pressure, that these pressures can simply be added and then converted into a combined flow. For naturally-induced infiltration (i.e., any combination of stack and wind effects) some areas of the envelope will see positive pressures and some will see negative pressures. If we are combining two flows, the areas of positive and negative pressures will, in general, be different for the two different driving forces. Thus, we would expect that VFE superposition would overpredict the total flow.

Since the exponent is never greater than unity, we can be assured that the combined flows will never be larger than the arithmetic sum (or less than the difference) of the individual flows. Thus, an expression (analogous to the law of cosines) can be derived to combine two flows:

$$Q^2 = Q_1^2 + Q_2^2 - \alpha Q_1 Q_2 \tag{10}$$

where $|\alpha| \leq 2$. We call this law *general quadrature.*

The quadrature constant, α, is undoubtedly not a universal constant; it depends on many of the details of the individual flows (e.g., exponent, leakage distribution, f_n, etc.). For each of the superposition laws described above, eq. 10 can be used to derive a value of the quadrature constant. Because these laws are all symmetric with respect to wind and stack, the quadrature constant will depend only on the ratio of the smaller flow to the larger one. Table 1 displays this data for two values of the air flow ratio.

Table 1: EQUIVALENT VALUES OF QUADRATURE CONSTANT (α)

(combined stack and wind at n=2/3)

f_n	*VFE*	*LBL*	*AIM*	*NRC*	*MITU*
1	-0.52	0	0.02	0.39	0.61
1/2	-0.49	0	0.08	0.31	0.43

In the appendix we derive some simplified expressions for these flows. The section below summarize these results and allow us to estimate numerical values as well as the strongest functional dependencies.

INFILTRATION-DOMINATED VENTILATION

The process of infiltration derives from pressure interactions across the building envelope. The distribution of these pressures will depend on the specifics of the driving forces. A useful and convenient quantity to consider is the leakage distribution angle, which quantifies the amount of leakage area under pressurization vs.

depressurization. The appendix of this report derives a generalized formulation for this relationship.

For any of the driving forces the induced air flow can be described in the following form:

$$Q = \mathbf{K} \left(\frac{\sin^{2n}\theta \cos^{2n}\theta}{\sin^{2n}\theta + \cos^{2n}\theta} \right) \Delta P^n \tag{11}$$

where the leakage coefficient and exponent are characteristic of the envelope, the leakage distribution angle and pressure are due to its interaction with the driving forces. The leakage distribution angle serves the function of quantifying how much of the external pressure contributes towards infiltration.

As derived in the appendix, the leakage distribution angle and external pressure for the driving forces under some reasonable assumptions can be summarized as follows:

Table 2: **PROPERTIES OF DRIVING FORCES**

Driving Force	*Distribution Angle*	*External Pressure* *
Stack (winter)	$\cos^2\theta_s = \beta$	$\lvert \Delta\rho \rvert \, gH$
Stack (summer)	$\sin^2\theta_s = \beta$	$\lvert \Delta\rho \rvert \, gH$
Wind (head on)	$\tan\theta_{\equiv} = 3^{-1/2n}$	$C'\rho v^2$
Wind (diagonal)	$\tan\theta_{\neq} = 1$	$C'\rho v^2$
Fan (Supply)	$\cos 2\theta_f = 1$	n/a
Fan (Exhaust)	$\cos 2\theta_f = -1$	n/a

Care was taken to carry the non-linearities (associated with the exponent) through the formulation. Although it appears in expressions for flows and leakage distribution angles, the exponent has very little effect on the value of the addition efficiencies and hence the superposition.

When two forces are acting together, a perturbation analysis can be used to estimate the interaction and derive an addition efficiency for the effect of the smaller force:

$$Q = Q_1 + \varepsilon_+ Q_2 \tag{12}$$

where

$$\varepsilon_+ \leq 1 \quad \text{and } Q_2 < Q_1 \tag{13}$$

As shown in the appendix (eq. 45) the addition efficiency can be expressed in terms of

* The external pressures are shown for reference only; the pressure drop typical of any given leak is approximately half this value. Their exact calculation and interpretation depends on the details of the infiltration model used and do not materially affect the superposition.

the leakage distribution angles of the two driving forces:

$$\varepsilon_+ = \left| \frac{\cos 2\theta_1 + \cos 2\theta_2}{2} \right| \tag{14}$$

These factors can be used for fan flows larger than the naturally-induced flow by applying a minimum value of the efficiency to convert ε_+ to ε_f.

These addition efficiencies are least robust when the two flows are of the same size (i.e., $f_n \approx 1$), because this is the regime in which the flow is most sensitive to the details of the leakage. Within the context of simple models, however, such uncertainty must be accepted.

Table 3 lists the addition efficiencies for different combinations of wind, stack, and (unbalanced) fan flow.

Table 3: **ADDITION EFFECIENCIES**

$\text{Wind}_{\neq}$ Effect + fan:	$\varepsilon_+ = 0.5$
$\text{Wind}_{\equiv}$ Effect + Exhaust fan:	$\varepsilon_+ = 0.16$
$\text{Wind}_{\equiv}$ Effect + Supply fan:	$\varepsilon_+ = 0.84$
Stack Effect(winter) + Exhaust fan:	$\varepsilon_+ = 1-\beta$
Stack Effect(winter) + Supply fan:	$\varepsilon_+ = \beta$
Stack Effect(summer) + Exhaust fan:	$\varepsilon_+ = \beta$
Stack Effect(summer) + Supply fan:	$\varepsilon_+ = 1-\beta$
Stack + $\text{Wind}_{\neq}$ Effects:	$\varepsilon_+ = \lvert \beta - .5 \rvert$
Stack(winter) + $\text{Wind}_{\equiv}$ Effects:	$\varepsilon_+ = \lvert \beta - .16 \rvert$
Stack(summer) + $\text{Wind}_{\equiv}$ Effects:	$\varepsilon_+ = \lvert .84 - \beta \rvert$

Note: *n=2/3 was used for the head-on wind effect,* $\text{Wind}_{\equiv}$.

The case for all three driving forces operating simultaneous can be treated as follows: In general, two of the driving forces must be combined and then an estimate of the resulting air flow and leakage distribution angle must be made to add the third. If, however, one of the driving forces dominates, it can be assumed that the leakage distribution angle remains unchanged after addition of one of the small ones to the large one.

Quadrature Constant

In the form of eq. 12 the expressions do not directly relate to quadrature, but we can put them into such a form by squaring the expression:

$$Q^2 = Q_1^2 + 2\varepsilon_+ Q_2\, Q_1 + \varepsilon_+^2 Q_2^2 \tag{15}$$

or, equivalently,

$$Q^2 = Q_1^2 + Q_2^2 + \left[2\varepsilon_+ + (\varepsilon_+^2 - 1) f_n\right] Q_1 Q_2 \tag{16}$$

Comparing this to eq. 10 and solving for α we obtain the following:

$$\alpha = f_n (1-\varepsilon_+^2) - 2\varepsilon_+ \tag{17}$$

for a combination of stack and wind.

These expressions also apply to natural flows plus a small fan. If the fan becomes larger than the natural flow, however, f_n must be replaced by f_f and ε_+ by ε_f, in order to account for fan domination. (See eqs. 46 and 47 in the appendix.)

We can use eq. 17 to derive numerical values of α for a few combinations of driving forces in Table 4. Table 4a combines the wind and the winter stack effect. Because eq. 14 is symmetric with respect to the two leakage distribution angles, table 4a (like table 1) does not differ depending on which natural driving forces is larger.

Table 4a: **TYPICAL VALUES OF α**
Combined Wind and (Winter) Stack Effects

	f_n	β=3/4	β=1/2	β=1/4
$\text{Wind}_{\equiv}$ + Stack	1	-0.53	0.20	0.81
	1/2	-0.85	-0.24	0.32
$\text{Wind}_{\neq}$ + Stack	1	0.44	1.00	0.44
	1/2	-0.03	0.50	-0.03

In these tables we have assumed that n=2/3, (which is only important for the head-on wind effect), and that winter conditions prevail. If the outside temperature is higher than the inside, the β=3/4 and β=1/4 columns should be interchanged. Table 4b contains the combination of the winter stack effect with fans.

Table 4b: **TYPICAL VALUES OF** α Combined (Winter) Stack and Fan Effects			
	SUPPLY + STACK		
f_f	β=3⁄4	β=1⁄2	β=1⁄4
2	-0.63	0.5	0.5
1	-1.06	-0.25	0.44
1/2	-1.28	-0.63	-0.03
f_f	β=1⁄4	β=1⁄2	β=3⁄4
	EXHAUST + STACK		

Note that for large fan flows the quadrature constant, becomes equal to the inverse of the the fan efficiency factor, (i.e., $\alpha \rightarrow 1/f_f$). Table 4c combines the wind with fans.

Table 4c: **TYPICAL VALUES OF** α Combined Wind and Fan Effects			
	WIND$_{\equiv}$		*WIND*$_{\neq}$
f_f	*Exhaust Fans*	*Supply Fans*	
2	0.5	-1.09	0.5
1	0.65	-1.38	-0.25
1/2	0.16	-1.53	-0.63

As discussed earlier the combination of all three driving forces can be done using pair-wise combinations, for a restricted set of assumptions.

DISCUSSION

It is instructive to compare the values of the quadrature constant calculated here, with the equivalent values of α from the literature, which predominantly focus on combining wind and stack effects. Thus we can compare the entries in Table 4a with the reported values from Table 1. There are two entries in Table 1 that are based on detailed measurements, the AIM dataset (Ref 15), or detailed simulations, the MITU dataset (Refs 10,13), of specific buildings. Our derivation may have some explanatory power for these entries.

The test houses in the AIM dataset are closely set in a row; thus, the wind effect could only contribute when the wind impacted directly (i.e., only the wind$_{\equiv}$ entries apply). Further these houses had little low leakage, but did have some high leakage; thus we would expect the neutral level to be in the range $\frac{1}{2} \leq \beta < \frac{3}{4}$. Thus the first two entries on the first line of Table 4a would be expected to (and do) bracket the AIM entry in Table 1. For this configuration the quadrature constant is consistent with zero.

The configuration for the MITU dataset was somewhat different. MITU was unshielded and completely exposed to the wind, whose speed and direction varied (i.e., both the wind$_{\equiv}$ and wind$_{\neq}$ entries apply). Further MITU had no high leakage, but had significant floor leakage into a crawlspace; thus we would expect the neutral level to be in the range range $\frac{1}{4}<\beta<\frac{1}{2}$. Thus the right two columns of Table 4a should best bracket the results for this dataset in Table 1. Both the calculations and data are consistent with a value of α for the MITU dataset of approximately one half.

Comparing Tables 1 and 4a again, it is clear that VFE superposition is only consistent with our model if the neutral level is quite high and the wind is head on. As these conditions are not typical, it is not surprising that the literature finds that such a superposition model overpredicts. Similarly, NRC superposition is consistent with either wind that strikes primarily on the diagonal or a low neutral level. The literature, however, does not contain enough details to carry this comparison further.

It is clear that an optimum value for the quadrature constant depends on the distribution of leakage and wind angle. Values in the range $-1<\alpha<1$ are not unreasonable. Often we do not have enough specific information about a structure to estimate the quadrature constant and so it would be useful to have a default value. If we assume that the default house has a slightly high neutral level, that we are interested in non-summer conditions and a majority of the wind effect comes from wind impinging directly on a surface, then simple quadrature (i.e., $\alpha=0$) is a good default. (For summer conditions $\alpha=\frac{1}{2}$ might be a better assumption.)

Addition of Fans

The discussion has focussed so far on the combination of wind and stack effects. Indeed, this has been the area of most interest over the last decade. As mechanical ventilation becomes a more important component in residential buildings, the need to accurately include the effects of fans increases.

Tables 4b and 4c contain the calculated values of the quadrature constant for the case when a (supply or exhaust) fan is added to either wind or stack flow. Although numerically equivalent, it is more useful when dealing with fans to talk about their addition efficiency, ε_f, rather than their quadrature constant.

An examination of the fan addition efficiencies in Table 3 leads one to the conclusion that in general (small, unbalanced) fans contribute approximately 50% of their actual flow rate towards increasing the total ventilation. Such efficiencies must be considered when making either energy or indoor air quality calculations. Furthermore, if we can assume this *50% rule* in general, then we can combine all three driving forces easily.

A comparison of simple quadrature and the fan efficiency shows that they never deviate by more than 10% of the total infiltration. Therefore, when combining natural (i.e., any combination of wind and stack) and fan-induced infiltration, simple quadrature is a good approximation, for the special case of the 50% rule.

The *50% rule* is good overall, but there are deviations depending on the distributions and which natural force dominates. Some of the important deviations can be summarized as follows:

- *When the wind dominates supply fans have a larger addition efficiency than exhaust fans.* The differential becomes larger as more of the wind strikes directly on a face. This effect may be especially important during shoulder seasons in which a small ventilation fan is being utilized for indoor air quality purposes.
- *For high neutral level houses, when the winter stack effect dominates, supply fans have a larger addition efficiency than exhaust fans.* This effect implies that for a house with many ceiling penetrations such as kitchen and bathroom exhausts, there may be less impact on total ventilation from running these fans during the winter than was thought. Thus a local exhaust fans in the winter may be energy efficient means of extracting a specific pollutant source (i.e., bathroom or stove), but will have a relatively small impact on whole-house ventilation.
- *For high neutral level houses, when the summer stack effect dominates, supply fans have a smaller addition efficiency than exhaust fans.* This effect may be important for the slab-on-grade house typical of the sun-belt of the United States and is the converse of the previous one.
- *For large fan flows the total air flow through the envelope becomes equal to the fan flow.* As the fan dominates the infiltration, the fan addition efficiency increases asymptotically to unity.

CONCLUSIONS

The concept of leakage distribution angle as developed in this report is critical to the understanding of how the forces that drive infiltration interact. The leakage distribution angle quantifies the partitioning between the areas of the envelope that infiltration and exfiltrate. Since the key factor in superposition is the separation of canceling and augmenting pressures, this partitioning allows a more fundamental description of the superposition process.

An examination of how pressure and leakage distributions interact across the envelope of a building has allowed us to develop some general guidelines for the superposition of stack, wind and fan effects without detailed modeling assumptions. We can summarize the work of the report in a single superposition equation which takes into account all three of these forces:

$$Q = \sqrt{Q_s^2 + Q_w^2 - \alpha Q_s Q_w} + Q_{balanced} + \varepsilon_f Q_f \tag{18}$$

The quadrature constant, α, depends on the leakage and wind angle distributions as well as the sign of the temperature difference. For any reasonable configuration

$$-1 < \alpha < 1. \tag{19}$$

When little is known about the details of the building, a default value of zero can be used; if some information is known, however, the estimate can be improved using the methods developed herein.

The fan addition efficiency, ε_f, indicates the contribution an unbalanced mechanical ventilation system has on the total ventilation:

$$0 < \varepsilon_f \leq 1 \tag{20}$$

When little is known about the details of the system, the *%50 rule* (i.e., a value of one half subject to an overall minimum of the fan flow) can be used as default. As this effect may depend strongly on the season and whether it is a supply or exhaust fan, care should be taken when estimating the impact of a particular mechanical system on the ventilation rate.

When both default values are used the superposition law becomes the following↓:

$$Q = Q_{balanced} + \sqrt{Q_s^2 + Q_w^2 + Q_f^2} \tag{21}$$

Without information on the neutral level, the wind direction and the relative dominance of wind and stack effects, this expression is the best general law of superposition for infiltration-dominated ventilation.

Although these expressions do not explicitly contain the exponent, the exponent was considered in their derivation. The value of the exponent has little to do with the form or result of the superposition equation, save through its effect on the leakage distribution angle. Under special circumstances the effect of the exponent on the superposition can be quite significant, but as the numerical impact of this effect is in general small, the exponent is not of critical importance to the issue of superposition. Superposition notwithstanding, the exponent has an appreciable impact on the individual flows. Future work will investigate the extension of the leakage-distribution-angle concept for the calculation of the individual flows and the change in the leakage distribution angle as two forces operate.

↓ When Modera and Peterson simulated the MITU configuration, they found the same result. As can be seen from table 4, this was caused by a cancellation of errors for low neutral level in the winter with an exhaust fan. Their result does, however, indicate that this expression may be useful in some circumstances when the default assumptions are not met.

APPENDIX: GENERALIZED CALCULATION OF INFILTRATION

For natural driving pressures such as wind and stack some of the building envelope will be under negative pressure and some of the building envelope will be under positive pressure. We can conceptually simplify the formulation by combining together all of the positive pressures into a single value and similarly for the negative ones, without having to know the details:

$$Q_{+} = \mathbf{K}_{+} \left(P_{+} - \Delta P_{o} \right)^{n} \tag{22.1}$$

$$Q_{-} = \mathbf{K}_{-} \left(\Delta P_{o} - P_{-} \right)^{n} \tag{22.2}$$

where

$$\mathbf{K} = \mathbf{K}_{+} + \mathbf{K}_{-} \tag{23}$$

Ignoring density difference* the infiltration and exfiltration must be equal

$$Q = Q_{+} = Q_{-} \tag{24}$$

So that

$$\Delta P_{o} = \frac{\mathbf{K}_{+}^{1/n} P_{+} + \mathbf{K}_{-}^{1/n} P_{-}}{\mathbf{K}_{+}^{1/n} + \mathbf{K}_{-}^{1/n}} \tag{25}$$

The infiltration can be rewritten as

$$Q = \frac{\mathbf{K}_{+}\ \mathbf{K}_{-}}{(\mathbf{K}_{+}^{1/n} + \mathbf{K}_{-}^{1/n})^{n}} \Delta P^{n} \tag{26}$$

where

$$\Delta P \equiv P_{+} - P_{-} \tag{27}$$

This suggests some useful definitions for the combined leakage

$$\mathbf{K}_{o} \equiv \left(\mathbf{K}_{+}^{1/n} + \mathbf{K}_{-}^{1/n} \right)^{n} \tag{28}$$

and the leakage distribution angle

$$\tan\theta \equiv \left(\frac{\mathbf{K}_{+}}{\mathbf{K}_{-}} \right)^{1/2n} \tag{29}$$

which is defined only in the first quadrant as the leakages are all non-negative. So that the infiltration can also be expressed as

$$Q = \mathbf{K}_{o}\ (\Delta P \sin^{2}\theta \cos^{2}\theta)^{n} \tag{30}$$

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* This density difference will be important for an exact calculation of the stack effect. For the purposes of this report we need not consider it.

Note that $\mathbf{K}_o$ is not the same thing as $\mathbf{K}$, but depends on the leakage distribution angle:

$$\mathbf{K}_o = \frac{\mathbf{K}}{\sin^{2n}\theta + \cos^{2n}\theta} \tag{31}$$

These expressions are applicable to any set of driving forces that operate on the outside of the envelope. As the fan effect does not effect the exterior pressure field, these expressions apply for only the *stack* and *wind* effects.

Stack Effect

If the densities of two bodies of air are different, there will be a gravity-induced pressure gradient between them. In buildings this density difference is caused by temperature differences and is know as the *stack effect*. We can approximate the pressure drop and leakage distribution angle for the stack effect as follows:

$$\Delta P_s \approx \left|\Delta\rho\right| g\,H \tag{32}$$

where H is the effective stack height of the building. Although the exact formulation of this pressure difference depends on leakage distribution and is beyond the scope of this report; the effective stack height, however, is on the order of the height of the building.

One simple approximation uses the (dimensionless) neutral level of building:

If the inside temperature is greater than outside:

$$\cos^2\theta_s \approx \beta \tag{33.1}$$

If the inside temperature is greater than outside:

$$\sin^2\theta_s \approx \beta \tag{33.2}$$

The neutral level, β, is that (non-dimensionalized) height at which the inside and outside pressures are equal when only the stack effect is in operation. The non-dimensionalization factor is the height from the lowest to highest leak.

EXAMPLE: As an example, assume that all of the leakage is at or near the floor and ceiling and that there is twice as much high leakage as low leakage. If the floor-ceiling height is H and the inside is warmer than the outside,

$$\cos^2\theta_s = \beta = \frac{2^{1/n}}{1+2^{1/n}} \tag{34.1}$$

$$Q_s = \mathbf{K}\left\{ \frac{1}{3}(1+\frac{1}{2}^{1/n})^{-n} \right\}\left(\Delta\rho\,g\,H\right)^n \tag{34.2}$$

where for typical values of exponent (i.e., n=2⁄3) the term in curly brackets is approximately 0.27.

Wind Effect

The wind effect acts by causing different pressure shifts on the faces of the structure. We can approximate the pressure drop and leakage distribution angle for the wind effect as follows:

$$\Delta P_w = C'\rho v^2 \tag{35}$$

where C' is the effective shielding coefficient of the structure. This coefficient is less than unity and gets smaller as the local shielding increases.

Both the shielding coefficient and the leakage distribution angle will be a function of wind-direction, building aspect ratio, and leakage distribution, and will not be developed herein.

The inside pressure coefficient normally is about -0.2 for evenly distributed leakage,[16] but may be anywhere between -1 and 1.

Head-On Wind

In the typical case in which the wind strikes one of the faces of the building head-on, we assume there is no air flow through the floor or ceiling, but all four walls have the same leakage.

$$\tan\theta_{\equiv} = \left(\frac{1}{3}\right)^{\frac{1}{2n}} \tag{36.1}$$

which for the typical value of the exponent (i.e., $n=2/3$)

$$\cos 2\theta_{\equiv} = 0.68 \tag{37.1}$$

Diagonal Wind

If the wind comes from a diagonal rather than head on, two sides of building will have positive pressures and two will have negative. Although the total infiltration will not change much the leakage distribution angle will— the tangent will become equal to unity or, equivalently,

$$\cos 2\theta_{\neq} = 0 \tag{38}$$

EXAMPLE: As an example we take the *head-on* case and further assume that the wind pressure coefficient for the windward side is 0.7 and for all other sides it is -0.5. We can then solve for wind-induced air flow:

$$Q_{\equiv} = \frac{\mathbf{K}}{4}\,(0.45\rho v^2)^n \tag{39.1}$$

$$Q_{\equiv} = \mathbf{K} \left\{ \frac{(1+\frac{1}{3}^{1/n})^{-n}}{4} \right\} \left[.45(1+(\frac{1}{3})^{1/n})\ \rho v^2 \right]^n \tag{39.2}$$

where for this example $C'_{\equiv} = 0.54$ and the factor in curly brackets is equal to 0.22.

Effect of Fans

The stack and wind effects operate by inducing a pressure on the outside surface of the envelope; the internal pressure responds to balance the flow. Mechanical ventilation (i.e., fans) does not effect the external pressure at all, rather it changes the internal pressure— inducing a change in the air flow through the envelope.

If the fan dominates the total ventilation, the flow will follow eq. 1. The fan flow may be affected by this pressure drop, in which case the fan curve should be used to determine the flow and pressure which simultaneously meets this equation and the fan curve. The operation of the fan either completely pressurizes or completely depressurizes the (vast majority of the) envelope, Thus, as we can see from eq. 29, the leakage distribution angle takes only one of two values depending on the fan direction.

$$\cos 2\theta_f = \begin{cases} 1 & \text{for supply fans} \\ -1 & \text{for exhaust fans} \end{cases} \tag{40}$$

If the fan does not dominate the ventilation there will be both infiltration and exfiltration through the envelope and the difference will be the fan flow:

$$\left| Q_+ - Q_- \right| = Q_f \tag{41}$$

The fan flow will then affect the infiltration and exfiltration by changing the internal pressure

$$Q_+ = \mathbf{K}_+ \left(P_+ - \Delta P_o + \delta P \right)^n \tag{42.1}$$

$$Q_- = \mathbf{K}_- \left(\Delta P_o - P_- - \delta P \right)^n \tag{42.2}$$

SMALL FAN FLOW

If we assume that the pressure shift δP is small, then the leakage distribution angle will not change. Further we can expand the power-law and solve for the pressure shift:

$$\delta P = \frac{Q_f}{n\mathbf{K}_o} (\Delta P \sin^2\theta \cos^2\theta)^{1-n} \tag{43.1}$$

$$\frac{\delta P}{\Delta P} = \frac{\sin^2 2\theta}{4n} \frac{Q_f}{Q_n} \tag{43.2}$$

where Q_n is the value of the naturally-induced infiltration that would have occurred without the fan (i.e., Q from preceding equations). It is sufficient for the fan flow to be small compared to the natural flow in order that the requirement for a small angular shift to be met.

This definition can be used to solve for the combined infiltration of a fan with a natural driving force:

$$Q = Q_n + \varepsilon_+ Q_f \tag{44.1}$$

$$\varepsilon_+ = \left| \frac{\cos 2\theta_f + \cos 2\theta}{2} \right| \tag{44.2}$$

LARGER FAN FLOWS

If the fan flow is not small, the distribution of pressures across the envelope will change in a complex way as a result of the fan. For the example of an exhaust fan, the fan will eventually increase the leakage distribution angle until all of the flow is infiltration at this point the total infiltration is equal to the fan flow and there is no exfiltration through the envelope. Once this point is reached the total air change is just equal to the fan flow.

The behavior when the natural and fan flows are comparable depends strongly on leakage distribution and exponent. As a simplification we assume that the small fan flow expression is applicable until the fan dominates:

$$Q = MAX\left(Q_f, Q_n + \varepsilon_+ Q_f \right) \tag{45}$$

Equivalently we could define a fan addition efficiency, ε_f to include this effect:

$$\varepsilon_f \equiv MAX\left(\varepsilon_+, 1 - \frac{1}{f_f} \right) \tag{46}$$

where the fan factor is defined similarly to f_n:

$$f_f \equiv \frac{Q_f}{Q_n} \tag{47}$$

In this discussion we have assumed that fans are constant flow devices. Although in reality some fans are pressure sensitive and the interaction of the fan with the envelope may cause them to significantly alter their behavior, the constant flow assumption is good in most cases. The pressure shift δP, however, may be used to estimate this effect from the fan curve if necessary.

Combining Two Natural Driving Forces

Unlike ideal fans, infiltration due to a natural driving force is very much effected by changes in the pressure distribution caused by another natural driving force.

We can use the formalism above to combine these two effects by treating the smaller of the two as a fan. However, since each one of the effects has both infiltration and exfiltration we must treat combine the effects of an exhaust fan coupled with infiltration (in which case the effects add) and a supply fan coupled with infiltration (in which case the effects partially cancel). If the distribution of positive and negative pressures from the two forces are uncorrelated, the combination can be represented as the leakage weighted difference between them:

$$Q = Q_1 + Q_2 \left| \left(\frac{1 - \cos 2\theta_1}{2} \right) \cos^2\theta_2 - \left(\frac{1 + \cos 2\theta_1}{2} \right) \sin^2\theta_2 \right| \qquad (48.1)$$

$$Q = Q_1 + Q_2 \left| \frac{\cos 2\theta_1 + \cos 2\theta_2}{2} \right| \qquad (48.2)$$

Note that this expression reduces to the fan flow expression when a fan distribution angle is inserted.

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